The Discreet Charm of Abstraction: Hyperspace Worlds and Victorian Geometry

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This paper is drawn from research for my next book project on Victorian spatiality. Its provisional thesis is that several nineteenth-century discourses that posit a structuring difference between "inside" and "outside"—domestic architectural theory, Euclidean geometry, and spiritualism—share an underlying impulse to establish sites of plenitude as compensatory bulwarks against loss, lack, or emptiness. My project traces the connections between notions of interiority in these technical disciplines and the exploration of subjective experience undertaken in Victorian aesthetic theory and fiction. What these disciplines share is a utopian, exteriorizing impulse: an attempt to mark off, often in quite literal terms, new spheres of privilege, meaning, and presence by banishing perceived threats to disciplinary and civilizing boundaries.

The unifying thread throughout my analysis is the rhetoric of privacy and domesticity. As I started reading the many late Victorian imaginative meditations and fables about other-dimensional worlds alongside debates over the epistemological implications of the new geometries in professional and popular periodicals, I noticed several

Abstract: Analytic or Cartesian-coordinate geometry, which describes space in terms of algebraic equations, was conceived by most Victorians as a translation of the "real-world" truths of Euclidean geometry. However, other commentators were concerned that once Euclid's axioms were translated into a purely symbolic system, there would be no guarantee that that system corresponded to reality. The epistemological crisis intensified with mathematical advances in the later decades of the century, as algebraic equations began to yield problems that seemed to require additional spatial dimensions for their resolution. Charles Howard Hinton, an important popularizer of hyperspace philosophy, argued that imperceptible higher-dimensional worlds must actually exist, since they are conceivable mathematically. The algebraic system is no longer merely meant to represent an a priori truth (of Euclidean space); it also reveals something about that space that is not accessible through perceptual means. This paper examines the anxiety attendant upon the threat of an abstract hyperreal in the work of three imaginative hyperspace writers: Gustav Theodor Fechner, Charles Hinton, and H. G. Wells.

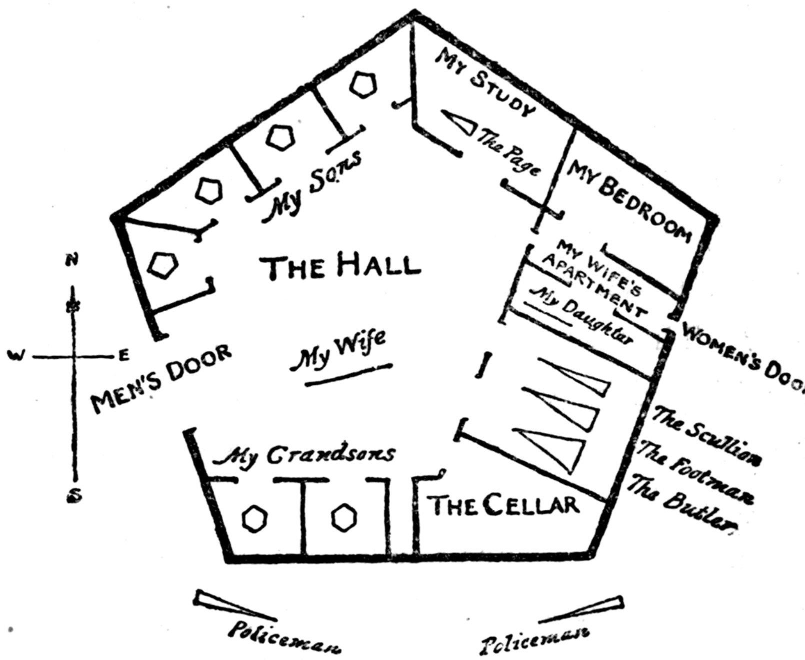


Fig. 1. A Flatland house, from *Flatland: A Romance of Many Dimensions* by Edwin A. Abbott (London: Seely, 1884): frontispiece.

recurrent features. First, an obsession with what the physical parameters of domesticity in other dimensions might look like; second, an anxiety about the effects that higher-dimensional space would have on the integrity and privacy of three-dimensional human bodies; third, a concern with questions of scientific evidence occasioned by multi-dimensional space; and finally, a utopian fantasy about the amplitude and wholeness of higher-dimensional worlds, which is strongly marked by nostalgia for a lost world of transparent information and self-evidence, a nostalgia occasioned by reason and thought experiment rather than sensory perception.

For example, Edwin Abbott's 1884 novella *Flatland*, which treats an imaginary world of two dimensions and the flat geometrical shapes who live there, relates how its narrator, A Square, is visited by an emissary from Spaceland, A Sphere, and comes to understand the limitations of his own perceptions. A Square is at great pains to describe the layout of Flatland houses, including the rigid compartmentalization of inhabitants by sex (see fig. 1). The ideology of separate spheres is literalized: privacy is secured not by the psychological internalization of gendered realms of propriety, but by the imposition of actual lines. After A Sphere

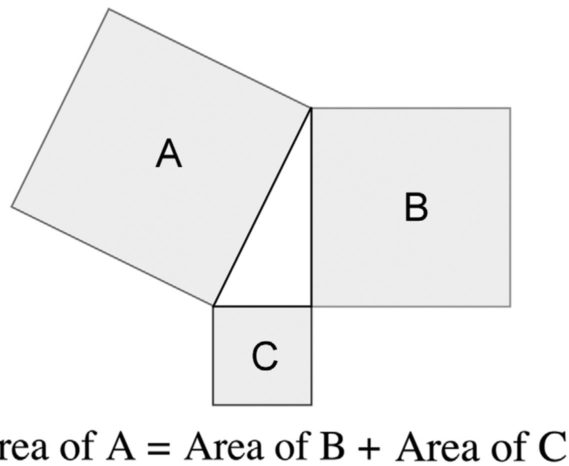
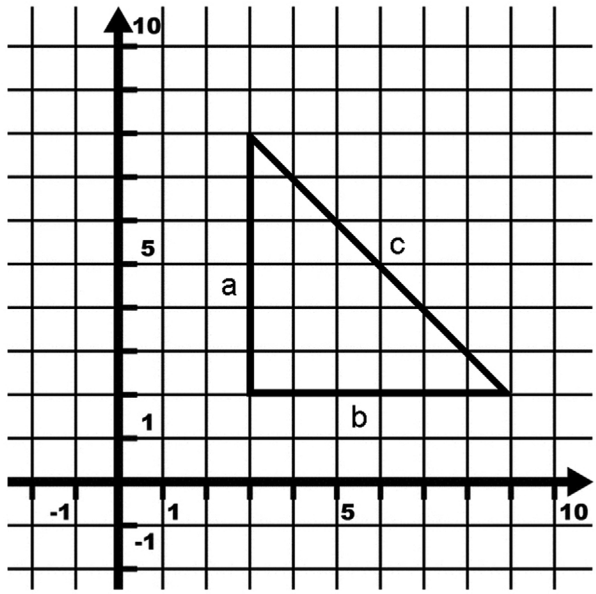


Fig. 2. The Pythagorean theorem expressed with shapes.

lifts A Square physically into Spaceland, they fly over Flatland and suddenly, to A Square's great consternation, the interior of his own house is laid open: "I looked below, and saw with my physical eye all that domestic individuality which I had hitherto merely inferred with the understanding. . . . All this I could now *see*" (96, original emphasis). A Square then continues to reason that there must be even higher dimensions, from the perspective of which there is no privacy for Spacelanders, either. Toward the end of their journey, A Square asks his three-dimensional friend:

> Let me beg thee to vouchsafe thy servant a sight of thine interior . . . thy stomach, thy intestines. . . . even as we, who are now in Space, look down on Flatland and see the insides of all things, so of a certainty there is yet above us some higher, purer region . . . from the vantage-ground of which we shall look down together upon the revealed insides of Solid things. . . . Doubtless we cannot *see* that other higher Spaceland now, because we have no eye in our stomachs. But . . . of a surety there is a Fourth Dimension, which [can be] perceive[d] with the inner eye of thought. (102–04, original emphasis)

Abbott and other higher-dimensional fabulists are responding directly to recent innovations in mathematics, specifically analytic algebra. To review briefly: Euclidean geometry treats the properties of shapes as they (arguably) exist in the real world (see fig. 2).[1] Analytic algebra, invented by René Descartes in the seventeenth century, expresses values using algebraic equations involving



$$c = \sqrt{a^2 + b^2}$$

Fig. 3. The Pythagorean theorem expressed with algebraic coordinates.

the coordinates of the points lying on the shape (see fig. 3). Once the seemingly self-evident truths of Euclidean geometry are translated into a purely symbolic and abstract system, a representational crisis occurs: what is the guarantee that the system corresponds to anything in reality? The British mathematician Augustus De Morgan was one of the first to express these concerns explicitly. According to historian Joan Richards, as early as the 1830s he "entertained the possibility that algebra might entail simply an internally consistent symbolic development which was more or less susceptible to any particular interpretation" (45). De Morgan's anxiety was echoed by William Whewell, who remarked in 1845 that "mere analytical reasoning is a bad discipline of the intellect, on account of the way in which it puts *out of sight* the subject matter of the reasoning" (45–46, my emphasis).

As long as analytic geometry confined itself to expressing truths that could also be represented using Euclidean geometry, however, and as long as the two systems continued to correspond, the crisis was dormant. Yet challenges to this correspondence were brewing. Algebraic equations were beginning to yield problems that seemed to demand the addition of another spatial dimension for their resolution. Several mathematicians began to suggest that the development of *n*-dimensional geometries, or hyperspace philosophy as it was called, might be necessary. As Richards emphasizes, these mathematicians assiduously avoided hinting at the actual reality of four- (or greater-) dimensional space (55–59). But by the time William Spottiswoode addressed the British Association for the Advancement of Science in 1878, the question of the reality of *n*-dimensional space was rather more mazy:

> Manifold space . . . is not seriously regarded as a reality in the same sense as ordinary space; it is a mode of representation, or a method which, having served its purpose, vanishes from the scene. Like a rainbow, if we try to grasp it, it eludes our very touch; but like a rainbow it arises out of real conditions of known and tangible quantities, and if rightly apprehended it is a true and valuable expression of natural laws. (22–23)

Higher-dimensional geometries (which are still Euclidean) can be conceived as extensions of "ordinary" three-dimensional geometry. The shape corresponding to the dimension *n* (for example, a two-dimensional square) is generated by a projection of the shape of dimension $n - 1$ (in this example, a one-dimensional line). So just as the three-dimensional cube is an extension into the third dimension of a two-dimensional square, the hypercube (or tesseract) is simply an extension into the fourth dimension of a cube. If each dimension is an extension, or spatial analogy, of the one before, then in theory one could continue mathematically to generate greater dimensional spaces ad infinitum.

In England the most important popularizer of *n*-dimensional geometries was Charles Howard Hinton, coiner of the term "tesseract," who expressed this analogical insight in mathematical terms:

> If there is a straight line before us two inches long, its length is expressed by the number 2. Suppose a square to be described on the line . . . this figure is expressed by the number 4, *i.e.*, 2×2. . . . generally written 2^2. . . . If on the same line a cube be constructed, the number of cubic inches in the figure so made is 8, *i.e.*, $2 \times 2 \times 2$ or 2^3. . . . The question naturally occurs, looking at these numbers 2, 2^2, 2^3, by what figure shall we represent 2^4. . . . ("What Is" 9–10)

By what figure shall we represent? Hinton's question attempts to argue backward from the terms of analytic algebra to the natural, "real" world. The Cartesian system is no longer meant to represent an a priori truth (of Euclidean space) translated into other language, but instead is taken as the ground of truth that must reveal something about that space that is not otherwise accessible to us through sensory or other means. As Elizabeth Throesch puts it, the "concept of the fourth dimension of space grew out of a slippage between the languages of these two forms of mathematics, a hypostatization of abstract symbols" (41). This phenomenon is precisely that described by Jean Baudrillard in *Simulacra and Simulation*:

> Today abstraction is no longer that of the map, the double, the mirror, or the concept. . . . It is the generation by models of a real without origin or reality: a hyperreal. The territory no longer precedes the map, nor does it survive it. It is nevertheless the map that precedes the territory . . . that engenders the territory. . . . But it is no longer a question of either maps or territories. Something has disappeared: the sovereign difference, between one and the other, that constituted the charm of abstraction. (1–2)

In the rest of this paper, I sketch out some of the themes that recur in the work of three hyperspace philosophers—Charles Hinton, Gustav Theodor Fechner, and H. G. Wells—along with some theoretical implications. Fechner was a German experimental psychologist who was arguably the first to write about four-dimensional space. His essays on the topic appeared in German in the 1840s and were translated into English in the 1870s, when they influenced Abbott, among others. Fechner's work established a rhetorical strategy that was taken up by many later theorists: namely, hectoring. As Fechner complains, most people reading his essay "Space Has Four Dimensions"

> will prefer the old three dimensions with their comfortable breadth and thickness in spite of all their inherent shortcomings . . . to the new dimension of sheer progress toward improvement. They will value the room in three-dimensional space that allows them to get out of the way of those who are dedicated to unconditional progress. Above all, they will be vexed by the question: Where in the world of one dimension will they find room for their belly and how thin must the sausages be, if the entire pig is only as thick as a (mathematical) line? (133)

Fechner's claim of "unconditional progress" is not a mere rhetorical feint:

> Our three-dimensional universe is an immense "primary sphere," which comprises a set of individual spheres. Each of these spheres moves in an orbit thus the primary sphere also moves, but where can it move if there is no fourth dimension? As the primary sphere moves through the fourth dimension, all the embedded spheres

> and all that is alive on them is carried through the fourth dimension. The foregoing opens the way for a beautiful observation. Everything that we will experience is already here, and all that we have experienced is still here. . . . Mankind is now spared all worry. All bread that he will eat has already been baked, he need not even open his mouth to eat it, his mouth will be opened for him once the world reaches a certain location, and a bit further his mouth will be closed. Any bruise which one will suffer has already been bandaged and has healed. The money that one will collect already lies counted and will only be collected in the crossing of three dimensions. . . . In short, henceforth mankind can live the most comfortable life in the world; it will always arrive where it is destined to arrive. (134–36)

For Fechner, importantly, this "recognition" entails nothing more than a mental adjustment. There are no experimental means by which we can access empirical knowledge of the fourth dimension; we can only reason our way—against all mental resistance—into an apprehension of its apparent truth. In Fechner and other hyperspace theorists the exhortation to belief is made in pragmatic terms: "The benefits of the fourth dimension are so great and so evident that there is absolutely no reason why it should not be accepted just as readily as other ideas that promised less" (Fechner 133).[2]

In the work of Hinton, this utopian strain is modified in a more recognizably scientific direction.[3] For him, exhortation is driven by a desire for a more complete rational understanding of the workings of the universe, which is presumably reward enough: "We have abandoned the simple and instinctive mode of life of the earlier civilisations for one regulated by the assumptions of our knowledge. . . . by supposing away certain limitations of the fundamental conditions of existence as we know it, a state of being can be conceived with powers far transcending our own" ("What Is" 3).

Yet just as in *Flatland*, where access to higher-dimensional worlds entails looking into houses and stomachs, Hinton's essays and fables focus on the mundane minutiae of domestic arrangements in other dimensions (see fig. 4). In his story "A Plane World" (1886), which treats two-dimensional figures living on a flat surface, heteronormative domesticity is all that is physically possible: "It is evident that the sharp point of one man is always running into another man's sensitive or soft edge. Each man is in continual apprehension of every other man. . . . In this land no such thing as friendship or familiar intercourse between man and man is possible" (145) (see fig. 5). But when

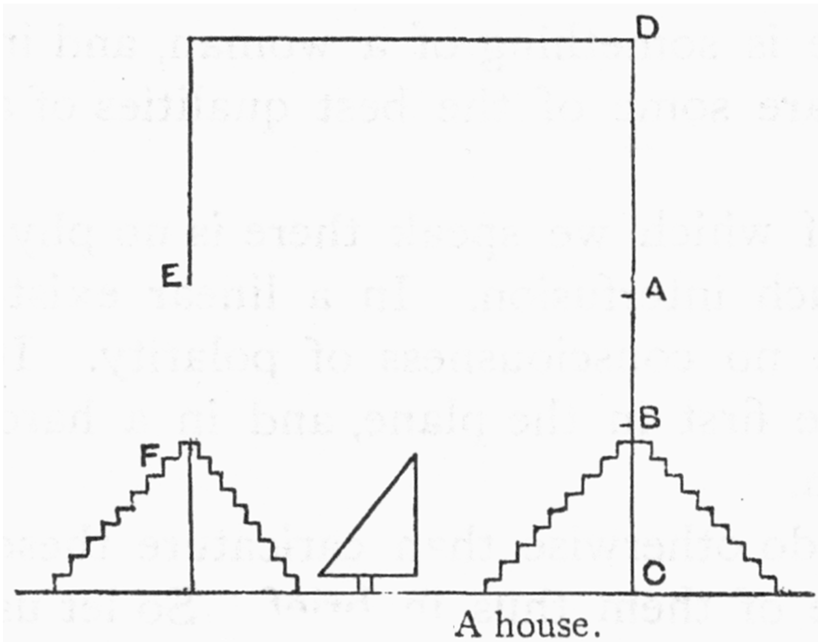


Fig. 4. A Plane World house, from "A Plane World" by C. H. Hinton (*Scientific Romances: First Series*, London: Merchant, 1886): 139.

> Homo and Mulier be placed together, a very different relationship manifests itself. They cannot injure one another and each is framed for the most delightful converse with the other. Nothing can be more secure from the outside world than a pair of approximately the same height; each protects the sensitive edge of each, and their armoured edges and means of offence are turned against all comers, either in one direction or the other. (146)

Perfect Victorian domestic bliss is secured by naturalized gender difference, reified in geometric form (see fig. 6).

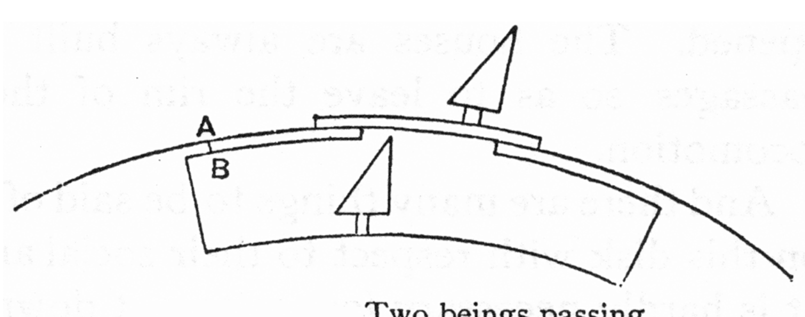


Fig. 5. Two beings passing in Plane World, from "A Plane World" by C. H. Hinton (*Scientific Romances: First Series*, London: Merchant, 1886): 139.

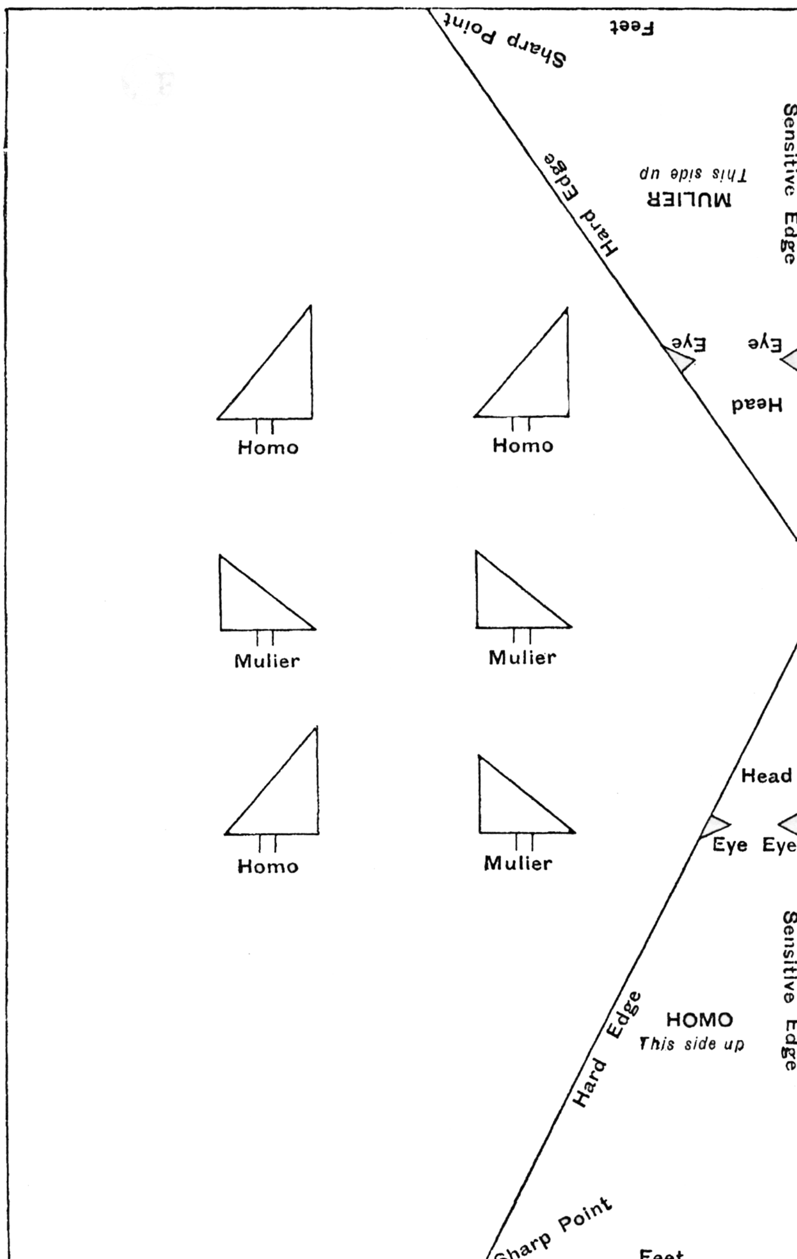


Fig. 6. Homo and Mulier, from "A Plane World" by C. H. Hinton (*Scientific Romances: First Series*, London: Merchant, 1886): 142.

Domesticity is threatened, however, by sudden access to higher-dimensional awareness: the story recounts a strange incident in which Mulier is “suddenly, in all outward respects, turned irremediably into a man” (146). Mulier has learned how to access a higher dimension (that is, the third), has lifted up off the plane surface of their world, and accidentally flipped over before re-descending. Of course she and her mate are distressed by this turn of events; “one day, with fear, she said that she would either die or be restored to the outward semblance of her sex” (147).

A similar state of affairs occurs in H. G. Wells’s “The Plattner Story” (1897). In it, a hapless master of modern languages has an accident with a mysterious green powder and is blasted into the fourth dimension. When he returns, he is also “flipped over,” as it were: all the organs inside his body are on the opposite sides, and he can now write only with his left hand, moving from right to left. Wells’s treatment of this world is more spiritual than Hinton’s. The four-dimensional world is peopled by ghostly, enormous-eyed, limbless figures with “the appearance of human heads, beneath which a tadpole-like body swung” (19), who are obsessively focused on the three-dimensional world Plattner has left behind. He, and they, can of course see through walls and move easily from place to place in a manner impossible in three dimensions. (One of the first things Plattner does is hover above his old classroom, where he sees his students cheating on their homework using a crib of Euclid.) The narrator surmises that these four-dimensional figures are the ghosts of the dead who have unfinished business with the living: “It may be . . . that, when our life has closed, when evil or good is no longer a choice for us, we may still have to witness the working out of the train of consequences we have laid” (23).

Yet the main action of the story takes place when Plattner is suddenly transported to a domestic scene peopled by strangers. One night he is wandering about in this new world when he is “arrested by the sight of the thing that was happening in a room in a back street. . . . The windows were open, the blinds up, and the setting sun shone clearly into it, so that it came out quite brightly at first, a vivid oblong of room, lying like a magic-lantern picture” (24). What he sees is a man dying in bed while a woman rustles through papers in a cabinet in the corner of the room. Plattner has visual access to such scenes, but no other contact is possible—no one can hear or see him, and figures pass through him unheeding as if he, or they, were ghosts. His role is observational; one might say omniscient-narratorial. Later the narrator informs us that he has exercised restraint in not embroidering this incident for dramatic purpose: "I have resisted, I believe successfully, the natural disposition of a writer of fiction to dress up incidents of this sort. . . . I have carefully avoided any attempt at style, effect, or construction. It would have been easy, for instance, to have worked the scene of the death-bed into a kind of plot in which Plattner might have been involved" (27–28).

For Hinton and other hyperspace theorists, the new knowledge attendant upon access to higher dimensions is marked by an insistent, yet impossible, visuality. (Wells is at great pains to point out that the "exoteric" [11] evidence for Plattner's tale was supplied by exactly six and a half pairs of eyes, and thus all he has told "is established by such evidence as even a criminal lawyer would approve" [12].) As Hinton surmises in the essay "What Is the Fourth Dimension?", for a four-dimensional being there could be "no barrier no confinement of our devising that would not be perfectly open to him. He would come and go at pleasure; he would be able to perform feats of the most surprising kind" (25). Yet an understanding of this state of affairs—if not the ability to see into the interiors of bodies and houses—is available to us three-dimensional types as well, through ratiocination: "There comes overpoweringly upon the mind of one, who thinks on higher space, the certainty that all we think, or do, or imagine, lies open. In that large world [of higher dimensions] our secrets lie as clear as the secrets of a plane being lie to an eye above the plane. . . . And so we lie palpable, open. There is no such thing as secrecy" (Hinton, "Many Dimensions" 42).

It is tempting to suggest that the last decades of Victoria's reign were particularly suited to nostalgia for a world of perfectly fitted geometric creatures forming a literal interiority, a barrier against outside encroachment. For these hyperspace writers, the gutting of privacy and bodily integrity—in other words, of domesticity—allows a greater understanding of the nature of the universe as fundamentally complete; nostalgia goes accompanied by a compensatory fantasy of the power of the same knowledge that eliminated secrecy. As Hinton ruminates, "Any one, who will try, can find that, by passing deeper and deeper into absolute observation of matter, and familiarity with it, that which he first felt as real passes away. . . . Thus there springs before the mind an idealism which is more real than matter; a glimpse of a higher world, which is no abstraction" ("Many Dimensions" 42). Hinton anticipates Baudrillard's lament about the disappearance of the "charm of abstraction" that was constituted by the difference between the territory and the map (or, we might say, between the ontological reality of *n*-dimensional space and the algebraic formulas that either bring it into being or were called into existence to represent it). As Baudrillard notes in a radically different context, the moment when "the whole body becomes a sign to offer itself to the exchange of bodily signs" (112) is enabled (perhaps even necessitated) by a cultural context in which "there is no longer a private and domestic universe, there are only incessant figures of circulation" (113). Yet for pre-post-modern hyperspace writers like Fechner, Hinton, Abbott, and Wells, the collapsing of map and territory unleashes utopian possibilities, such as knowledge of the spirit world and a deep understanding of the universe as replete, rather than signaling the decay of all belief in an authentic extratextual referent.

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NOTES

[1]I say "arguably" since Euclidean representations of objects are themselves ideal, abstractions that cannot exist in the "real world" (for example, there can be no actual triangle whose angles are exactly equal).

[2]The question of Fechner's position in this essay is a tricky one. The translators note that in writing under the pseudonym "Dr. Mises" Fechner signals his satirical intent, although the object of his satire is most likely claims of scientific certainty rather than the idea of the fourth dimension itself, in which he was quite prepared to believe. Whether the utopian content should be ascribed to Fechner himself or to other writers whom he here mocks, my point remains that a strong utopian strain runs throughout higher-dimensional writings—a strain that will be developed even further in the work of Hinton.

[3]Since drafting this essay I have become aware of Mark Blacklock's essay "'On the Eve of the Fourth Dimension': Utopian Higher Space," which examines Hinton's longer work *A New Era of Thought* (1888).

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